- Formal definition of a **limit**: Let $f(x)$ be a function defined on an open interval that contains $x = a$, except possibly at $x = a$ itself. Then, $\lim_{x \to a} f(x) = L$ iff for every $\varepsilon > 0$, there is some number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \varepsilon$.
- Main ways to go about evaluating a limit:
  - Plug it in.
  - Factor and cancel.
  - Rationalize or use conjugates.
  - Squeeze Theorem.
  - Take the logarithm.
    - You can also pull out the logarithm (or any other operator) if necessary.
  - L'Hôpital's Rule (to be introduced later).
- Limits can be broken up into its individual components of sums and factors.
- **One-sided limit**: The limit as we approach $x = a$ from only one side of the function.
  - If $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$, then $\lim_{x \to a} f(x)$ does not exist.
- **Squeeze theorem**: If $g(x) \leq f(x) \leq h(x)$ on a closed interval containing $x = a$ (except possibly $x = a$ itself) and $\lim_{x \to a} g(x) = L = \lim_{x \to a} h(x)$, then $\lim_{x \to a} f(x) = L$.
- **Continuity**: There is continuity at $x = a$ if $\lim_{x \to a} f(x) = f(a)$ (i.e. function = limit)
- **Infinite limits**:
  - Plug it in.
  - If the function is the quotient of two polynomials, then divide by x to the degree of the polynomial in the denominator to evaluate the limit as $x \to \pm\infty$.
  - If the function is periodic, the limit does not exist as $x \to \pm\infty$
  - If there is a vertical asymptote at $x = a$, then one-sided limits as $x \to a$ is either positive or negative infinity.
  - End behavior: evaluate the limit of $f(x)$ as $x \to \pm\infty$
  - Use them to find asymptotes.

Additional ideas:

- Squeeze theorem can be used to prove certain trigonometric limits, especially $\lim_{x \to 0} \frac{\sin x}{x} = 1$ and $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$. This can be used with trigonometric summation properties to prove $\frac{d}{dx} \sin x = \cos x$ and $\frac{d}{dx} \cos x = -\sin x$.